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THE LOGIC OF EINSTEIN'S EQUATION IN BOOLEAN ALGEBRA AND THE COMPATIBILITY THEORY

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Abstract

This study assessed whether Einstein's equation, $E=mc^2$, could retain its meaning when the logical equivalence sign, $\stackrel{T}{\equiv}$, replaced the equality sign,=, based on Boolean algebraic system. In other words, we analyzed whether $E\stackrel{T}{\equiv}mc^2$ could mean the same thing as $E=mc^2$.The study thus addressed the possible logical content of Einstein's equation and juxtaposed the logic, $E\stackrel{T}{\equiv}mc^2$, with the maths and/or physics, $E=mc^2$, declaring them compatible. Here, we questioned whether $E\stackrel{T}{\equiv}mc^2$ means the same thing as $E=mc^2$. The result was, that **logical equivalence based on Boolean equality suggests that energy is logically equivalent to mass x the speed of light squared, $E\stackrel{T}{\equiv}mc^2$.** Thus, being equivalent, we declared the compatibility of $E\stackrel{T}{\equiv}Mc^2$ with $E=Mc^2$.

Key words: Compatibility Theory, Einstein's Equation, Boolean Algebra, Equivalence

Question: Whether $(E\stackrel{T}{\equiv}Mc^2) \stackrel{T}{\equiv} (E=Mc^2)$?

Introduction

Compatibility theory states that compatibility exists if units are equivalent, or approximates, or similar, or intersected, or complements (Essien: 2008; 2011c). Hence, the formula: C=**{(≡)|(≈)|(~)|(∩)|(c)}**,where C is compatibility,

$\equiv$ is equivalence,

$\approx$ is approximation,

$\sim$ is similarity,

$\cap$ is intersection,

c is complement.

The compatibility theory presents equivalence, approximation, similarity, intersection and complementation as the basic conditions for compatibility between or among units and systems. The statements, $E \overset{T}{\equiv} mc^2$, energy is logically equivalent to mass x the speed of light squared, and $E=mc^2$, energy equals mass x the speed of light squared, could be found to be compatible.

Einstein's equation, $E=mc^2$, signifies the equivalence of mass and energy, the most important upshot of the special relativity theory (Einstein, 1919). It postulates that the laws of physics are the same in all inertial frames of reference. The other postulate holds that the speed of light in vacuum is always measured to be the same no matter how fast the source or observer may be moving (Einstein, 1996, 13). By the first postulate, that is, the laws of physics being invariant in all inertial frames of reference, energy and mass are equivalent. Hence $E=mc^2$, where E is energy, m is mass and c is the speed of light.

Since this study shall be mainly a logical analysis of the equation, we first need to understand the meaning of the symbols E and m. For most physicists and philosophers, E represents the total energy of a physical system S. The symbol m represents the relativistic mass of S, which is the mass of S as measured by an observer O that moves with a constant velocity v

relative to *S*. The equation demonstrates that energy can be converted into mass, and vice versa (Young and Freedman, 2004, 1403).

Physicists and philosophers have offered various interpretations and analyses of the equation. Some of these are Eddington (1929); Torretti (1996); Bondi and Spurgin (1987); Rindler (1977); Lange (2001, 2002); and Fernflores (1998, 1999). These scholars have preoccupied themselves with the questions of whether mass and energy are the same and whether mass is convertible into energy. However, in this study, we shall address the question whether the mathematical equality in the equation, $E=mc^2$, when replaced with logical equivalence, $\overset{T}{\equiv}$, can make energy and mass equivalent.

Statement of the Problem

In mathematics equality is the state of being quantitatively the same. Ontologically, equality is sameness. Logically, equality is defined as identity. The equality sign (=) and the logical equivalence sign ($\overset{T}{\equiv}$) are used to indicate identity. Do they mean the same thing when used for relations such as the Einstein's relation of energy and mass? Suppose that in the equation, $E=mc^2$, that energy is the same with mass, can Energy retain its sameness with mass if the equation were $E\overset{T}{\equiv}mc^2$?

This problem differs from the three main philosophical questions about the interpretation of the equivalence of mass and energy that have occupied philosophers and physicists, reported by Fernflores (2010):

1. Are mass and energy the same property of physical systems and is that what is meant by asserting that they are "equivalent"?

2. Is mass "converted" into energy in some physical interaction, and if so, what is the relevant sense of "conversion"?
3. Does $E=mc^2$ have ontological consequences, and if so, what are they?

Some views by philosophers and physicists concerning these questions shall be presented here.

Review

As earlier indicated above, the three main philosophical questions about the interpretation of the equivalence of mass and energy that have occupied philosophers and physicists are:

1. Are mass and energy the same property of physical systems and is that what is meant by asserting that they are "equivalent"?
2. Is mass "converted" into energy in some physical interaction, and if so, what is the relevant sense of "conversion"?
3. Does $E=mc^2$ have ontological consequences, and if so, what are they?

To these questions philosophers and physicists have offered divergent opinions. Eddington in *Space, Time, and Gravitation* (1929) and Torretti in *Relativity and Geometry* (1996) are of the opinion that mass and energy are the same property of physical systems. Therefore, there is no sense in which one of the properties is ever physically convertible.

Eddington says: "it seems very probable that mass and energy are two ways of measuring what is essentially the same thing, in the same sense that the parallax and distance of a star are two ways of expressing the same property of location" (1929, 146). It is Eddington's position that the distinction between mass and energy is artificial, probably due to our use of different units to measure.

Torretti also observes that mass and energy seem to be different properties because they are measured in different units. He argues: “If a kitchen refrigerator can extract mass from a given jug of water and transfer it by heat or convection to the kitchen wall behind it, a trenchant metaphysical distinction between the mass and the energy of matter does not seem farfetched” (1996, 307, fn.13). According to Torretti, interactions where there appears to be a conversion of mass into energy (say) are really interactions where the distribution of the one property, call it “mass-energy,” changes (Fernflores, 2010). What Torretti insists on is that the apparent difference between mass and energy is an illusion which arises from “the convenient but deceitful act of the mind which we abstract time and space from nature” (1996, 307, fn.13).

Although Eddington’s and Torretti’s position (which this study upholds) that mass and energy are the same, they were, however, not occupied by the question which occupies me in this study, that is, the question of whether Einstein’s equation can be logically equivalent.

In “Energy has Mass” (1987), Bondi and Spurgin hold, however, that mass and energy are distinct properties and that there is no such thing as the conversion of mass and energy. Bondi and Spurgin (1987) argued that Einstein’s equation does not entail that mass and energy are the same property any more than mass and volume are the same. Bondi and Spurgin argue that, just as in the case of mass and volume, mass and energy have different dimensions. Bondi and Spurgin opine that the purported conversion of mass and energy is best understood as a transportation of energy. Fernflores (2010) criticizes Bondi and Spurgin that their interpretation fails to address reactions such as the electrons-positron annihilation reaction. This, according to Fernflores, implies that not only is the number of particles not conserved, but all of the particles involved are, by hypothesis, indivisible wholes. Thus, the energy liberated in such reactions cannot be explained as resulting from a transformation of the energy that was originally possessed by the constituents of the reacting particles. Bondi and Spurgin differ from my research interest in this study. First, they see mass as different from energy absolutely. Secondly, they are not occupied by the logic of the equation.

Rindler, in *Essential Relativity* (1977) agrees with Bondi and Spurgin that there are many purported conversions that are best understood as mere transformations of one kind of energy into a different kind of energy. According to Rindler's argument, there is nothing within special relativity itself that rules out the possibility that there exist fundamental, structureless particles (i.e, particles that are "atomic" in the philosophical sense of the world). If such particles exist, it is possible according to Einstein's equation that some or all of the mass of such particles "disappear" and an equivalent amount of energy "appear" within the relevant physically system. This seems to imply that we should confine our interpretation of mass energy equivalence to what we can deduce from special relativity. According to Fernflores, Rindler suggests that we should hold that Einstein's equation at least allows for genuine conversions of mass into energy, in the sense that there may be cases where a certain amount of inertial mass "disappears" from within a physical system and a corresponding amount of energy appears (Fernflores, 2010). Rindler's interpretation is mainly about the question of conversion of mass into energy. My study focuses, however, on the logic of Einstein's equation.

In "The Most Famous Equation" (2001) in *An Introduction to the Philosophy of Physics* (2002), Lange argues that rest-mass is the only real property of physical systems. In other words, there can be no such a physical process by which mass is converted into energy. Lange asks: in what sense can mass be converted into energy when mass and energy are not on a par in terms of their reality?" (2002, 227). There is no physical process by which mass is ever converted into energy. Instead, Lange argues, the apparent conversion of mass into energy (or vice versa) is an illusion that arises when we shift our level of analysis in examining a physical system.

My study asks the question whether the mathematical equality of Einstein's equation is the same with my proposed logical equivalence of the same equation, logically rendered as $E\overset{T}{\equiv}mc^2$. My study adopts the same property interpretation of the Einstein's equation. The reason is that equality and logical equivalence can operate on identity and not on *difference*. In other words, the mathematics and the logic involved in the Einstein's equation have made the researcher to adopt the same property interpretation of $E=mc^2$, especially Torretti's version, because it maintains the internal logic of relativity, which is the unity of space-time.

Furthermore, it is good to note that, here, in this study, the researcher is dealing with logical and mathematical processes, not physically processes. And one wonders, if this at all resembles Lange's analysis of the equation. Lange is of the view that there is no physical conversion of mass into energy. Although one implication of Lange's interpretation is that the apparent conversion could be a logical one, my study shifts focus from Lange's analysis and occupies itself with the question of whether the Einstein's equation could retain its status and meaning if it were rendered as $E\overset{T}{\equiv}mc^2$.

Logical equivalence based on Boolean equality suggests that energy is logically equivalent to mass x the speed of light squared, $E \overset{T}{\equiv} mc^2$.

Below, we shall make a logical ‘excursus’ into Boolean algebra. Although our focus is on logical equivalence, we cannot do without the basic operators, which are foundational to the derived operators; and the material equivalence, that is, the Boolean equality, in turn, serves as the basis for logical equivalence and the replacements. Logical equivalence is a tautologous material equivalence, a tautologous Boolean equality.

Boolean Algebra

Just like Aristotle with his disciple, Theophrastus, established the science of logic in ancient time, George Boole and Augustus De Morgan were pathfinders of modern logic. Modern logic was greatly developed, thereafter, by Gottlob Frege, Bertrand Russell and Alfred North Whitehead, unto set theory. In this section we shall treat of basic and derived logical operators as developed by George Boole. This involves Conjunction, Negation, Disjunction, Material Implication and Material Equivalence or Boolean Equality. We shall also indicate the distinction between a material equivalence and logical equivalence. We first of all indicate the meaning of statement variable and truth value.

A statement variable holds the place of a statement. It is a variable which is used in place of statements. Lower case letters such as x,y,z, or p,q,r, are used as variables. They can be used to stand for or signify any complete statement. For example, the statements,

Ama is a Ghanaian

can be represented by the statement variable “x”. The variable “x”, following a logical convention, can be used as the first variable. An argument containing many statements, such as:

All human beings are mortal

Ikpaha Eduok is a human being

Therefore Ikpaha Eduok is mortal

would be symbolized as:

x

y

.'. z

respectively.

The idea of truth value involves the assumption that every statement is either true or false. The idea of the law of excluded middle restricts us to estimate every statement as being either true or false. In propositional calculus, therefore, there are two possibilities for any statement, namely, its truth or falsity. Since every statement is either true or false, then every statement has a truth value. The truth value of a true statement is true, while the truth value of a false statement is false, excluding any middle-way possibility of being true and being false at the same time.

In English grammar, a simple statement is one that does not contain any other statement as a component. An example of a simple statement is: **“Abuja is neat”.** A compound statement is one that contains another statement as a component. An example of a compound statement is: “**Abuja is neat and Abuja is the capital of Nigeria**”. **“Abuja is neat”** is a simple statement.

"**Abuja is the capital of Nigeria**" is another simple statement. When put together, they form the compound statement, **"Abuja is neat and Abuja is the capital of Nigeria"**. Symbolic logic makes use of both simple and compound statements. However, there is more dominance or prevalence of the use of compound statements.

Boolean Operations

There are six Boolean operators (basic and derived) and five logical connectives based on their functions. Basic operators are so called because they form the bases for the derived operators; and the derived operators actually derive their foundations from the basic operators.

The **Basic Operators** are:

Negation	~
Conjunction	.
Disjunction (**Inclusive or**)	v

Derived Operators, which are composed from basic operators are:

Material implication	→
Exclusive **Or**	
Material equivalence	≡

Since negation does not connect two or more statements, it is not a logical connective.

The logical connectives, therefore, are:

Conjunction	.
Disjunction	v
Exclusive **Or**	⊕

Material implication	→
Material equivalence	≡

They are called logical connectives because they connect simple statements in a compound statement.

Negation (Not)

Any affirmative statement is negated if the word "**not**" is introduced into it. Let us suppose that the statement,

"Abuja is neat"

is true. This statement can be negated by saying,

"Abuja is not neat".

The logical symbol for negation is a curl or tilde "~ ". Using the variable 'x' to symbolize "Abuja is neat", the negation of "Abuja is neat", that is, "Abuja is not neat" becomes ~x. When "~" is used in a statement which was originally true, the result is a false one, and when it is used in a false statement, the result will be a true statement. In terms of truth value, "T" stands for true and "F" stands for false; and True (T) and False (F) are represented with the binary digits, 1 and 0, respectively. "~T" is "F" and "~F" is "T". In the truth value table we have:

X	~ x
T	F
F	T

With the binary digits,1 and 0, we have:

X	~ x

1	0
0	1

Conjunction (And)

A compound statement expressing conjunction links two statements by the word "and". In the statement:

"Abuja is neat and Abuja is the capital of Nigeria",

the two simple statements are conjuncts. Conjuncts are therefore the components of a conjunction. The statement, "Abuja is neat" is the first conjunct, while the statement, "Abuja is the capital of Nigeria" is the second conjunct. The dot (.) or an inverted vee (^) is used to symbolize "and". In this study we use the dot (.). Thus the conjunction, "Abuja is neat and Abuja is the capital of Nigeria" becomes "Abuja is neat . Abuja is the capital of Nigeria". Using the variable "x" to stand for "Abuja is neat" and the variable "y" to stand for "Abuja is the capital of Nigeria", the original statement becomes "x and y", which symbolically reduces to "x.q".

As a rule, **a conjunction requires both conjuncts to be true for the entire conjunction to be true**. The rule here in conjunction is that when both conjuncts are true, the conjunction is true, otherwise it is false. Representing a conjunction graphically in a truth-table we have:

X	y	x.y
T	T	T
T	F	F
F	T	F

F	F	F

With the binary digits, 1 and 0, we have:

X	y	x.y
1	1	1
1	0	0
0	1	0
0	0	0

Conjunction says that when:

x is true and y is true, then x.y is true

x is true and y is false, then x.y is false

x is false and y is true, then x.y is false

x is false and y is false, then x.y is false

Disjunction (Or) and Exclusive Or

The disjunction or alternation of two statements is formed by inserting the word "or" between them. The components of a disjunction are called "disjuncts" or "alternatives". An example of a disjunction is:

"Either Maame is sick or Maame has gone to Accra"

The symbol for disjunction is the wedge, "v", which is the initial letter of the Latin word "vel" which stands for "or". Using the variable "x" to stand for "Maame is sick" and the variable "y" to stand for "Maame has gone to Accra", the above statement may be symbolized as: x v y.

There are two senses of disjunction, namely, the weak or inclusive disjunction and the strong or exclusive disjunction. We have the **inclusive or** and the **exclusive or.**

The weak or inclusive disjunction is true in case one or the other or both disjuncts are true; and false if only both disjuncts are false. The **inclusive or** has the sense of "either, possibly both" true disjuncts for the disjunction to be true.

In a strong disjunction or **exclusive or**, the meaning of "or" is not "at least one" but "at least one or at most one, but not both". Where a restaurant lists "garri or fufu" on its lunch menu, it is clearly meant that, for the stated price of the meal, the lunch may have one or the other, but not both. In inclusive disjunction at least one of the statements in true and can both be true; while in exclusive disjunction, at least one of the statements is true but cannot both be true. For example,

1. **Either Maame is sick or She has gone to Accra.**
2. **Araba is looking after the mother in the hospital or at home preparing her dinner.**

In the **example 1** above, Maame could be sick yet she could still go to Accra. This example demonstrates the **inclusive or**, where at least one or both disjuncts can be true. **The example 1** given above, that is, either Maame is sick or Maame has gone to Accra" is an inclusive disjunction.

Example 2 illustrates the **exclusive or**, where both statements cannot be true. At least one or at most one statement can be true, but not both, for **how could Araba be looking after her mother in the hospital and be at home preparing her dinner (at the same time)**?

The rule in exclusive or is that we have truth value only when at least one or at most one alternative is true. When both are true or when both are false, it is false. This is a complement of equivalence which is true when both statements are true or when both are false.

The inclusive or is often symbolized as xvy. And with the rule regarding inclusive disjunction, that **at least one disjunct must be true to have a true disjunction**, we have the following truth table:

X	Y	Xvy
T	T	T
T	F	T
F	T	T
F	F	F

With the binary digits, 1 and 0, we have:

X	Y	Xvy
1	1	1
1	0	1
0	1	1
0	0	0

We have truth table for **Exclusive or** as follows:

X	Y	X⊕y
T	T	F
T	F	T
F	T	T
F	F	F

With the binary digits, 1 and 0, we have:

X	Y	X⊕y
1	1	0
1	0	1
0	1	1
0	0	0

It should be noted that the word “unless” can be used to form the disjunction of two statements.

Conditional or Material Implication ($\rightarrow$)

Two statements combined by placing the word “if” before the first and inserting the word “then” between them, result in a compound statement called a conditional statement or hypothetical statement or implicative statement or simply implication. The compound statement that follows the “if” is called the “antecedent” or “implicans “or” “protasis”, while the compound statement that follows the "then" is called the "consequent" or "implicate" or "apodosis". Let us consider the example below:

"If Manchester United wins the match then they will win the title".

"Manchester United wins the match" is the antecedent, while "they will win the title" is the consequent.

One of the symbols for conditional is the arrow " $\longrightarrow$". Using the arrow sign, the above example, "If Manchester United wins the match then they will win the title", we have the following notation:

$x \longrightarrow y$.

A conditional statement rules that we cannot have a true antecedent with a false consequent. In other words, if the antecedent is true, and the consequent is false, then the implication is false. As soon as there is a true antecedent and a false consequent, the implication is false. With this rule in mind, we have the following truth table:

x	y	$x \longrightarrow y$
T	T	T
T	F	F
F	T	T
F	F	T

With the binary digits, 1 and 0, we have:

x	y	$x \longrightarrow y$

1	1	1
1	0	0
0	1	1
0	0	1

Query

How can a false antecedent materially imply a true consequent thus making the implication true and how can a false antecedent materially imply a false consequent thus making the implication true? We can observe that in the 3rd and 4th rows that we have true implications with false consequents. How? This oddity can be obviated in the following mathematical analysis:

If $1<2$ then $1<4$

(If 1 is less than 2, then 1 is less than 4).

$1<2$ is true and $1<4$ is true.

For the apparent oddities in the third and fourth rows, we have the following examples to justify them:

If $3<2$ then $3<4$

(If 3 is less than 2 then 3 is less than 4). We know $3<2$ in false and $3<4$ is true; and this applies to the third row.

To justify the 4^{th} row, we have: If $4<2$ then $4<4$

(If 4 is less than 2 then 4 is less than 4). We know that 4<2 in false and that 4<4 is false; and this applies to the 4th row.

Bi-Conditional or Material Equivalence or Boolean Equality

Material equivalence is the truth functional connective which asserts that the statement it connects has the same truth value, that is, both true and both false. Material equivalence means that the two statements are both true and both false. In other words, we have true material equivalence when both statements are true and when both statement are false. If "true" and "true", then true; if "false" and "false", then true. Thus, "If and only if" is associated with material equivalence. "If and only if" can explain why material implication is called bi-conditional, since there are two "if's", for one "if" belongs to conditional.

The logical symbol for material equivalence is the triple-bar sign "≡". For two statements that are materially equivalent, we can write x≡y. The statement,

"The student will be awarded his degree if and only if he fulfills all the university requirements", can be symbolized as:

$$x \equiv y$$

What this means is that the student being awarded his degree is same as his having fulfilled all the university requirements. On the other way round, his having fulfilled all the university requirements is the same as his being awarded his degree, for he fulfilled all the university requirements.

With the rule regarding material equivalence that both statements must have same values for the material equivalence to be true, we present the following truth table:

x	Y	x ≡ y
T	T	T
T	F	F
F	T	F
F	F	T

With the binary digits, 1 and 0, we have:

x	Y	x ≡ y
1	1	1
1	0	0
0	1	0
0	0	1

Boolean equality or material equivalence means both statements are both true and both false.

x	Y	x ≡ y
1	1	1
1	0	0
0	1	0
0	0	1

Logical Equivalence or Tautologous Boolean Equality

There is a slight distinction between material equivalence and logical equivalence. Material equivalence means that both statements are both true and both false. Statements that are materially equivalent do not mean that they can be substituted for one another. The statements "Nigeria is larger than Ghana" and "Lome is the capital of Togo" are materially equivalent because they are both true, but one cannot replace the other.

When one statement replaces another, we have logical equivalence. Two statements that can replace one another are logically equivalent. This means that these statements have the same truth value and are also equivalent in meaning. There will not be and there cannot be any case in which one of these statements is true while the other is false. Two statements are logically equivalent when the statement of their material equivalence is a tautology. For being tautologically materially equivalent, there is a small "T" immediately above the triple bar "≡", such that "$\overset{T}{\equiv}$" symbolizes logical equivalence. Logical equivalence is thus a tautological material equivalence. Logical equivalence carries with it the idea of replacement. This means one statement can replace the other.

Rules of Replacement or Equivalence

The rules of replacement permit us to infer from any statement the result of replacing any component of that statement by any other statement logically equivalent to the component replaced. We remember that logically equivalent statements means that they have the same truth value, that is, both true and both false, and that they are also equivalent in meaning, capable of replacing one another. The rules of replacement are:

Double Negation (D. N.)

$$p \equiv \sim\sim p$$

The principle of double negation states that: "Any statement can be replaced with its double negation, and any doubly negated statements can be replaced with the statement".

Given $A \rightarrow B$, we can infer $A \rightarrow \sim\sim B$

Given $\sim\sim (AvB)$, AvB can be inferred by this rule.

De Morgan's Theorems (De M)

$$\sim (pvq) \equiv (\sim p . \sim q)$$

$$\sim (p.q) \equiv (\sim p\ v \sim q)$$

De Morgan's rule has two versions. The first rule, $\sim (pvq) \equiv \sim P. \sim q$ states that, "The negation of a disjunction can be replaced with a conjunction by dropping the negation sign, replacing the disjunction with conjunction, and negating each of the resulting conjuncts and vice versa". The second rule, $\sim (p.vq) \equiv (\sim pv\sim q)$ states that: "The negation of a conjunction can be replaced with a disjunction by dropping the negation sign, replacing the conjunction with a disjunction, and negating each of the resulting disjuncts and vice versa".

Given $\sim (AvB)$, we can infer $(\sim A . \sim B)$;

Given $\sim (\sim A . \sim\sim B)$, AvB can be inferred.

Commutation (Com.)

$$(pvq) \equiv (qvp)$$

$$(p.q) \equiv (q.p)$$

The idea of commutation is, that the order in which conjunctions and disjunctions are written does not impinge upon their truth value.

Given (A.B) → (C.D), → (B.A), (D.C) can be inferred
Given (AvB → (CvD), → (BvA), (DvC) can be inferred.

$$[(r.q).r] \equiv [p.(q.r)]$$
$$[(pvq) \vee r] \equiv [pv(qvr)]$$

Association (Assoc.)

Association claims that the truth value of conjunction and disjunction is not affected by the grouping of their components, except in a conditional. The rule holds for conjunction and disjunction but does not hold for conditional.

Distribution (Dist.)

$$[p. (qvr)] \equiv [(p.q) \vee (p.r)]$$
$$[p \vee (q.r)] \equiv [(pvq). (p \vee r)]$$

It should be noted that in both the original expression and its logical equivalent, that the same logical connective appears first.

Material Implication (M.I)

$$(p \rightarrow q) \equiv (\sim pvq)$$

It should be noted in this rule that conditionals and disjunctions can be substituted for one another.

Given $A \rightarrow B$, $\sim A \vee B$ can be inferred.

Given $A \rightarrow (B \vee C)$, $\sim A \vee (B \vee C)$ can be inferred.

Transposition (Trans.)

$$(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$$

Transposition permits the antecedents and consequents of conditionals to be interchanged. However, there is only change in the values of the components that are interchanged because (p q) is not logically equivalent to (q p). According to this rule, if (~ A B) is given, we can infer (~B A); and if (~ A ~ B) in given, we can infer (B A).

Material Equivalence

$$(p \equiv q) \equiv [(p \rightarrow q) . (q \rightarrow p)]$$

$$(p \equiv q) \equiv [(p.q) \vee (\sim p . \sim q)]$$

Material equivalence rule has two versions. The first version takes our minds back to the idea underlying material equivalence; that it results from the joint assertion of two conditionals. The second version takes our minds back to the rule governing truth table construction for material equivalence, which is, that "p ≡ q" is true if and only if either p and q are both true or both p and q are false. The formula (p.q) v (~ p . ~ p) is equivalent to stating p ≡ q.

Exportation (Exp.)

$$[(p.q) \rightarrow r] \equiv [p \rightarrow (q \rightarrow r)]$$

The rule of exportation shows that p and q are the antecedents of r in both its original statement and its equivalence.

Tautology (Taut.)

$p \equiv (p \vee p)$

$p \equiv (p . p)$

The truth conditions for disjunctions and conjunctions are applied in the two versions of tautology respectively.

Einstein's Equivalence Principle

In 1905, Albert Einstein revolutionized classical physics and turned physics upon its modern course. This revolution lay in the principle of relativity, which guarantees that all the laws of physics are the same in all inertial reference frames. Before he exalted this principle in the domain of modern physics, Einstein took into consideration the features of Newtonian mechanics, the Galilean transformation, Maxwell's electromagnetic theory of light, the Michelson-Morley experiment, and the Lorentz transformation, which are stepping stones in the special relativity theory.

Newtonian mechanics and gravitation, which exalted absolute space and time, relied on the Galilean transformation. The Galilean transformation assumes that time and length are absolute and their assumptions of absoluteness of time and length formed the foundation of Newtonian physics.

James Clerk Maxwell unified the laws of electricity and magnetism in a single law now referred to as “the Maxwell equation”, which is his theory of electromagnetism. A changing magnetic field induces an electric field while changing electric field in turn induces a magnetic field. Maxwell was unifying Faraday’s law, Gauss’ law (electric and magnetic) with Ampere’s law. However, the Galilean transformation is untenable for phenomena like electricity, magnetism or phenomena the velocities of which approximate the speed of light (c=3x108m/s).

The Michelson-Morley experiment, conducted in 1887, was designed to demonstrate the existence of the special frame of reference called the “ether” frame, and to determine the motion of the earth with respect to that frame. The result was that the velocity of light is the same when measured along two perpendicular axes in a reference frame which is moving relative to the ether frame in different times of the year (Alozie, 2003, 78). Einstein was to annul the ether frame and to see it as irrelevant and inapplicable to modern physical theorizing.

With the Maxwell’s equation and the result of the Michelson-Morley experiment, which indicated the propagation of light, Einstein postulated that the velocity of light is independent of the motion of its sources. This indicates invariance of the speed of light.

The Lorentz transformations perceive space and time in a symmetrical manner. What is a pure space interval or pure time interval in one reference frame becomes mixture of both space and time intervals in another reference frame. One consequence of the Michelson-Morley experiment was the special theory of relativity.

The special relativity theory which applies to systems in uniform motion was based on two postulates. First, the laws of physics are the same in all inertial frames. By this law, mass and energy are equivalent. Hence E=mc2 (where E is energy, m is mass and c is the speed of light). An inertial reference frame is a coordinate system in which the law of inertia applies. The earth can be regarded as an inertial frame if its motion is neglected. The laboratory in which we perform our experiment and which is fixed on earth is also an inertial frame, and so is a vehicle moving with uniform velocity relative to the laboratory.

The second postulate of the special relativity theory is that speed of light in vacuum is always measured to be the same no matter how fast the source or observer may be moving (Einstein, 1996, 13). The special theory of relativity deals with problems involving frames of reference in uniform relative motion. With a view to ascertaining the connection and interdependence of space coordinates with time, we shall see how the general theory of relativity treats problems involving frames of reference accelerated with respect to one another.

Meanwhile, the unity of spacetime, which holds sway in relativity theory, was realized by Minkowski. This was expressed in the opening words of his famous lecture on “Space and Time” as follows, that the views of space and time which wish to lay before you have sprung from the soil of experimental physics, and therein lies their real strength. They are radical. Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadow and only a kind of the two will preserve an independent reality. Minkowski was thus significant in the relativity theory as a conceptual scheme (Hamdan, 2007).

The application of the law of gravity to the principle of relativity gave rise to the general theory of relativity. The special principle of relativity is the principle of the physical relativity of all uniform motion. It holds that the laws of nature are invariant in all uniformly reference frames. The general principle of relativity upholds that all bodies of reference are equivalent for the description of natural phenomena, whatever may be their state of motion (Einsteing, 1996, 61).

The general theory of relativity arose through the extension of the principles of relativity to the gravitational field. Gravitation, in Einstein's understanding, was a field area of the material world, like electromagnetism. The development of the general theory of relativity was a result of generalization of experimental facts, such as the equivalence of inertial and gravitational mass. During the study of the properties of gravitation, Einstein stressed that:

> In contrast to electric and magnetic fields, the gravitational field exhibits a most remarkable property, which is of fundamental importance for what follows. Bodies which are moving under the sole influence of a gravitational field receive an acceleration, which does not in the least depend either on the material or on the physical state of the body (1996, 64).

To illustrate this, a piece of lead and a piece of wood fall in exactly the same manner in a gravitational field (in vacuo), when they start off from rest or with the same initial velocity. This law, Einstein holds, can be expressed in a different form in the light of the following consideration (1996).

By virtue of Newton's second law of motion, we have (force) = (Inertial mass) X (acceleration), where the "inertial mass" is a characteristic constant of the accelerated body. If new gravitation is the cause of the acceleration, we then have (force) = (gravitational mass) x (intensity of the gravitational field), where the “gravitational mass” is likewise a characteristic constant for the body. From these two relations follows:

(acceleration) = gravitational mass x (intensity of the gravitational inertial mass field)

Einstein argues that, if now, as we find from experience, the acceleration is to be independent of the nature and the condition of the body and always the same for s given gravitational field, then the ratio of the gravitational to the inertial mass must likewise be the same for all bodies, and by a suitable choice of units (Einstein Relativity 65). We then have the equivalence of gravitational mass of a body to its inertial mass. By gravitation, bodies attract each other with force that depends on the masses of their bodies and the distance between them.

The general theory of relativity explains gravitation through the dependence of its structure on the distribution and motion of masses of matter. Distribution and motion of masses of matter determine the difference between the structures of curvature-tensor space-time of the general theory of relativity. In its turn, this structure determines the motion of masses under the impact of the gravitational forces.

Thus, the masses of matter determine the structure of space-time, and determine their own movement. The difference of the structure of space-time from the flat metric, the field of the curvature, is the gravitational field. Discovery of the fact that the mass of bodies determines the geometric structures of time and space indicated the existence of an organic link between time, space, and matter. While this link was determined in the special theory of relativity solely by external material factors (it depended on the relative position and movement of the material bodies), in the general theory inner connections were discovered and it was shown that the metrics of the space-time continuum depended on the distribution of matter in the universe.

At the basis of the general theory of relativity is the idea that gravity and accelerated motion are fundamentally equivalent. This notion is known as the principle of equivalence. To illustrate this, following Einstein, who explained most of relativity through thought experiment, consider people in a closed elevator, who cannot see out. On the Earth's surface, gravity is a downward force. But even if they were out in space in a location where there is no gravity, there would still be an identical force in one direction as the wall or floor pushes on them if their space craft accelerated forward in the opposite direction at the proper rate. Einstein's principle of equivalence holds that there is no way that people in the elevator, without contact with the outside, could tell whether such a pull came from gravity or from an acceleration.

In the vicinity of a massive object, there would be a force toward the object; the force would be in a different direction depending on where we were located with respect to the massive object. We could, following the principle of equivalence, consider ourselves to be in a curved space accelerating down a slope toward the massive object instead of being subject to the object's gravity. Thus we can explain gravity as a curvature of space.

As earlier indicated in Einstein's argument, if the acceleration of a body is to be independent of the nature and the condition of the body and always the same for a given gravitational field, then the ratio of the gravitational to the inertial mass must likewise be the same for all bodies. The gravitational mass of a body is thus equivalent to its inertial mass

(Einstein, 1996, 65). Einstein acknowledged that this principle had hitherto been recorded in mechanics, but had not been interpreted.

Einstein explains it better. The same quality of a body manifests itself according to circumstances as “inertia” or as “weight”. The gravitational mass of an object is the weight of the object as measured on a balance scale. Gravitational mass is the measure of how much force the gravity of the earth exerts on an object. Newton’s laws describe the effects of this force, which vary with the distance of the mass from the earth. The second type of mass is inertial mass. This is the measure of the resistance of an object to acceleration. Here in context, inertial mass and gravitational mass are equal. This explains why a feather and a cannonball fall with equal velocity in a vacuum. The cannonball has hundreds of times more gravitational mass than the feather (it weights more) but it also has hundreds of times more resistance to motion than the feather (its inertial mass). Its attraction to the earth is hundreds of times stronger than that of the feather, but then so is its inclination not to move. The consequence is that it accelerates downward at the same rate as the feather, although it seems that it should fall much faster. In a nutshell, Einstein’s principle of equivalence refers to the equivalence of gravity and acceleration.

Juxtaposition

Placing the logic side by side with the mathematical physics, we here declare that $E \overset{T}{\equiv} mc^2$ is logically equivalent to $E=mc^2$. Hence, $E \overset{T}{\equiv} mc^2 \overset{T}{\equiv} (E=mc^2)$.

Conclusion

The compatibility theory affirms that entities are often compatible by their common property and yet by implicate order when they appear dissimilar. The order in each unit of a system is defined by the order of the system. In other words, the wholeness of a system is the common property and the implicate order which exist in the system. Compatibility theory thrives on the premise that entities which share a common property are often compatible and that there is compatibility by implicate order in entities that appear to be dissimilar. Entities, units or systems share common property if they are equivalent, or approximates, or similar, or intersected, or complements. Hence, $C=\{(\equiv)|(\approx)|(\sim)|(\cap)|(c)\}$, where compatibility entails equivalence, or approximation, or similarity, or intersection, or complement.

There are thus three interdependent postulates of the compatibility theory (Essien, 2008; 2011), as follows:

Postulate 1: The order in each unit of a system in relation to the order in another or other unit(s) within the same system is the order of the system.

Postulate 2: There is compatibility by implicate order.

Postulate 3: The all are in the one and the one is with the all, such that a set is defined only by the elements that it contains.

The compatibility theory, which presents equivalence, approximation, similarity, intersection and complementation, as the basic conditions for compatibility between or among units and systems, finds the statements, $E \overset{\top}{\equiv} mc^2$, that is, energy is logically equivalent to mass x the speed of light squared, and $E=mc^2$, that is, energy equals mass x the speed of light squared, to be compatible on the ground of the equivalence condition. Hence, $E \stackrel{\top}{\equiv} mc^2 \stackrel{\top}{\equiv} (E=mc^2)$.

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